6.4 Videos Guide

6.4a

- Calculus of the natural logarithmic function
  - $\frac{d}{dx}[\ln|x|] = \frac{1}{x}, \qquad x \neq 0$
  - $\int \frac{1}{x}\, dx = \ln|x| + C$

Exercise:

- Show that $\int \tan x \; dx = \ln|\sec x| + C$.

6.4b

Exercises:

- Differentiate the function
  - $f(x) = \ln(\sin^2 x)$
  - $h(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$
  - $y = \ln(\csc x - \cot x)$

6.4c

Exercises:

- Evaluate the integral.
  - $\int_0^3 \frac{dx}{5x+1}$
  - $\int \frac{\cos x}{2+\sin x}\, dx$

6.4d

- Calculus of logs and exponentials with bases other than e
  - $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$
  - $\frac{d}{dx}(b^x) = \ln b \cdot b^x$
  - $\int b^x\, dx = \frac{b^x}{\ln b} + C$

6.4e

Definition: (Euler's number e)

- $e = \lim_{x \to 0}(1 + x)^{1/x}$
- $e = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n$

6.4f

Exercise:

- Use logarithmic differentiation to find the derivative of the function.

$$y = \frac{e^{-x}\cos^2 x}{x^2 + x + 1}$$

6.4g

Exercise:

- Use logarithmic differentiation to find the derivative of the function.
  $y = x^{\cos x}$
- Process for logarithmic differentiation
  - Take the natural log of both sides
  - Use properties of logs
  - Differentiate (usually with respect to x)—use implicit differentiation for y
  - Give $\frac{dy}{dx}$ or y' in terms of x (or whatever the independent variable is)
- Summary of techniques involving exponents
  - $(\text{variable base})^{\text{constant exponent}}$ –power rule (ex: $y = x^2$)
  - $(\text{constant base})^{\text{variable exponent}}$ –rules for exponentials (ex: $y = e^x$ or $y = 2^x$)
  - $(\text{variable base})^{\text{variable exponent}}$ –logarithmic differentiation